[07-09-09C-T10]

Quadratic inequalities

■ **[EX1] Find all values of x for which $x^2 + 5x + 10 < 4$ is true.**

Solution.

$x^2 + 5x + 10 < 4$

$\Longrightarrow x^2 + 5x + 6 < 0$

$\Longrightarrow (x + 3)(x + 2) < 0$

Recall that the product of two numbers is negative when one of the numbers is positive and the other is negative. In this example, the two ways in which $(x + 3)(x + 2)$ will be negative are:

$(x + 3 < 0$ and $x + 2 > 0)$ OR $(x + 3 > 0$ and $x + 2 < 0)$.

Thus,

Case 1. $x + 3 < 0$ and $x + 2 > 0$

$x + 3 < 0 \Longrightarrow x < -3$ and $x + 2 > 0 \Longrightarrow x > -2$

$x \in (-\infty, -3) \bigcap (-2, \infty +) = \emptyset$

or

Case 2. $x + 3 > 0$ and $x + 2 < 0$

$x + 3 > 0 \Longrightarrow x > -3$ and $x + 2 < 0 \Longrightarrow x < -2$

$x \in (-\infty, -2) \bigcap (-3, \infty +) = (-3, -2)$

Since Case 1 $\Longrightarrow x \in \emptyset$ and Case 2 $\Longrightarrow x \in (-3, -2)$ and either Case 1 or Case 2 is true,

$x \in \emptyset \bigcup (-3, -2)$

$\therefore$ x∈(-3,-2).

■ **[EX2] Find all values of x for which $x^2 + 5x + 10 > 4$ is true.**

Solution.

$x^2 + 5x + 10 > 4$

$\Longrightarrow x^2 + 5x + 6 > 0$

$\Longrightarrow (x+3)(x+2) > 0$

Recall that the product of two numbers is positive when both of the factors are positive <u>or</u> when both the factors are negative. In this example, the two ways in which $(x+3)(x+2)$ will be positive are:

$(x + 3 > 0$ and $x + 2 > 0)$ OR $(x + 3 < 0$ and $x + 2 < 0)$.

Thus,

Case 1. $x + 3 > 0$ and $x + 2 > 0$

$x + 3 > 0 \Longrightarrow x > -3$ and $x + 2 > 0 \Longrightarrow x > -2$

$x \in (-3, \infty +) \bigcap (-2, \infty +) = (-2, \infty +)$

or

Case 2. $x + 3 < 0$ and $x + 2 < 0$

$x + 3 < 0 \Longrightarrow x < -3$ and $x + 2 < 0 \Longrightarrow x < -2$

$x \in (-\infty, -3) \bigcap (-\infty, -2) = (-\infty, -3)$

Since Case 1 $\Longrightarrow x \in (-2, \infty +)$ and Case 2 $\Longrightarrow x \in (-\infty, -3)$ and either Case 1 or Case 2 is true,

$\therefore\ x \in (-\infty, -3) \bigcup (-2, \infty +)$

Note that $(-\infty, -3) \bigcup (-2, \infty +)$ is all real numbers except those in $[-3, -2]$, so we could have written the answer as $x \in \mathbb{R} - [-3, -2]$.